

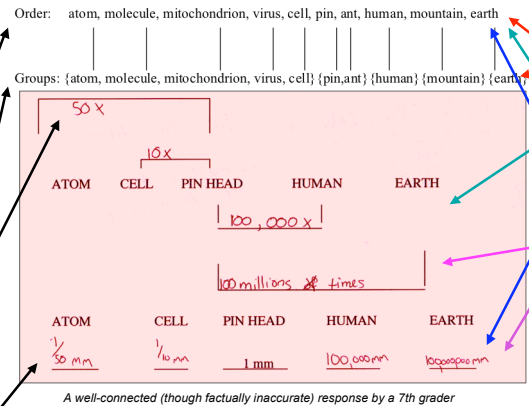
Development of a Learning Progression for Students' Conceptions of Size and Scale

1. INTRODUCTION

Learning progressions describe students' learning of a scientific idea over several years. They can guide curriculum, instruction, and assessment. (NRC, 2007)

2. FOUR CONCEPTIONS OF SIZE

- **Ordering** (Seriation): "the product of a set of asymmetrical transitive relations connected in series" (Inhelder & Piaget, 1969, pp. 5-6)
- **Grouping** (Classification): "implies a relation of resemblance between members of the same class, and one of dissimilarity between members of different classes." (Inhelder & Piaget, 1969/1959.)
- **Number of Times Bigger or Smaller** one object is than another: "The measurement process is... 'choose a unit, compare that unit to the object, and report the number of units.'" (unit is another object.) (NCTM, 2000/1989, p. 104)
- **Absolute Size:** As above, with conventional unit (e.g., 1 mm)



3. CONNECTED KNOWLEDGE

- **Order-Group Consistency:** Groups include contiguously-ranked objects.
- **Order-Times Bigger or Smaller Consistency:** Factor is larger for atom than cell, and for earth than human
- **Order-Absolute Size Consistency:** Sizes are larger for objects ranked bigger.
- **Times Bigger or Smaller-Absolute Size Consistency:**
 - Student realizes the connection between size and times bigger (conceptual).
 - The number of times bigger one object is than another, and the sizes of the two objects, are consistent (procedural). This is a matter of ratios and proportions.
- The focus of this study is on the content-independent connections between conceptions of size.

4. RESEARCH QUESTION

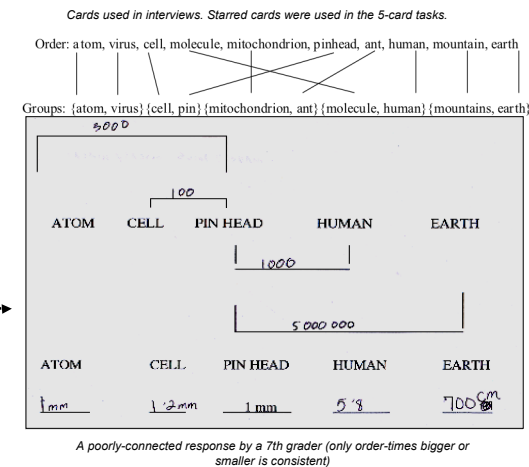
How well, when, and in what order do students through experts link the four conceptions of size?

5. PARTICIPANTS

- 41 students grades 7-12 from diverse small midwestern city public school district, ~50% free or reduced lunch.
- 6 undergraduates at midwestern research university

7. RESULTS

- Students' conceptions of size are often not well-connected.
 - Example: This 7th grader has inconsistent order-group, order-size (see sizes of cell, atom), and times bigger-size (atom is 3000 times smaller than the 1 mm pinhead, but atom is 1 mm in diameter.)
- The conceptions of size are connected in sequence, following a learning progression.
 - 45 of 47 students fit along this learning progression
 - Position along the progression is predicted by science course and academic ability ($R^2=0.48$); race and gender are not significant.



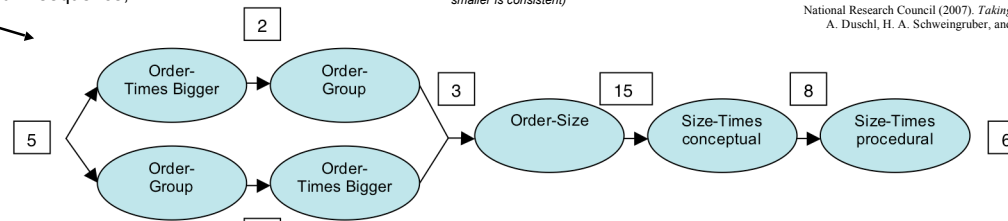
6. METHODS

Individual Interviews:

- Order the cards by size. Group the objects by size; make as many groups as you think makes sense. (10 cards)
- How many times bigger or smaller is each object than the head of a pin? What size is each object? (Size of pinhead given: 1mm) (5 cards*)

8. FUTURE DIRECTIONS

- Future analyses will focus on students' content knowledge, and how it relates to this learning progression.



A learning progression for size and scale
Number of students at each position is shown.